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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, OCTOBER 2019

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT01C04—GRAPH THEORY

(2012—2018 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*

Each question carries weight 1.

1. How many orientations does a simple graph of m edges have ?
2. Determine the connectivity and edge connectivity of the Petersen graph.
3. Give an example of a graph with n vertices and $n-1$ edges that is not a tree.
4. Prove that G is connected and unicyclic if and only if for some edge e of G, $G-e$ is a tree.
5. Describe covering of a graph G with an example.
6. Does there exist an Eulerian graph with an even number of vertices and an odd number of edges? Justify.
7. State Euler's formula. What are it's major consequences ?
8. Is the Petersen graph planar ? Justify.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*

Each question carries weight 2.

9. Prove that a simple cubic (3 – regular) connected graph G has a cut vertex if and only if it has a cut edge.
10. Prove that a simple graph G is connected if $\delta \geq \frac{n-1}{2}$.

Turn over

11. Prove that for a simple connected graph G, L (G) is isomorphic to G if and only if G is a cycle.

12. For any graph G, prove that $\left\lceil \frac{n}{1+\delta(G)} \right\rceil \leq \gamma(G) \leq n - \delta(G)$.

13. Prove that a connected graph is Eulerian if and only if each of its edge cuts has an even number of edges.

14. Show that a subset S of V is independent if and only if V \ S is a covering of G.

15. Prove that a graph is planar if and only if it is embeddable on a sphere.

16. Show that every graph with at most three cycles is planar.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. (a) In any graph G ; the number of vertices of odd degree is even.

    (b) Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G if and only if there exist vertices u and w of G distinct from v such that v is in every $v - w$ path in G

18. Prove that every connected graph contains a spanning tree.

19. Prove that the number oedges in a tree on n-vertices is $n - 1$, conversely, a connected graph on n vertices and $n - 1$ edges is a tree.

20. Prove that a graph G is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G

21. Prove that all plane embeddings of a given planar graph have the same number of faces.

22. Prove that a graph G is planar if and only if each of its blocks is planar.

(3 × 5 = 15)